ECS455 Chapter 2
Cellular Systems

2.2 Co-Channel Interference

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Office Hours:
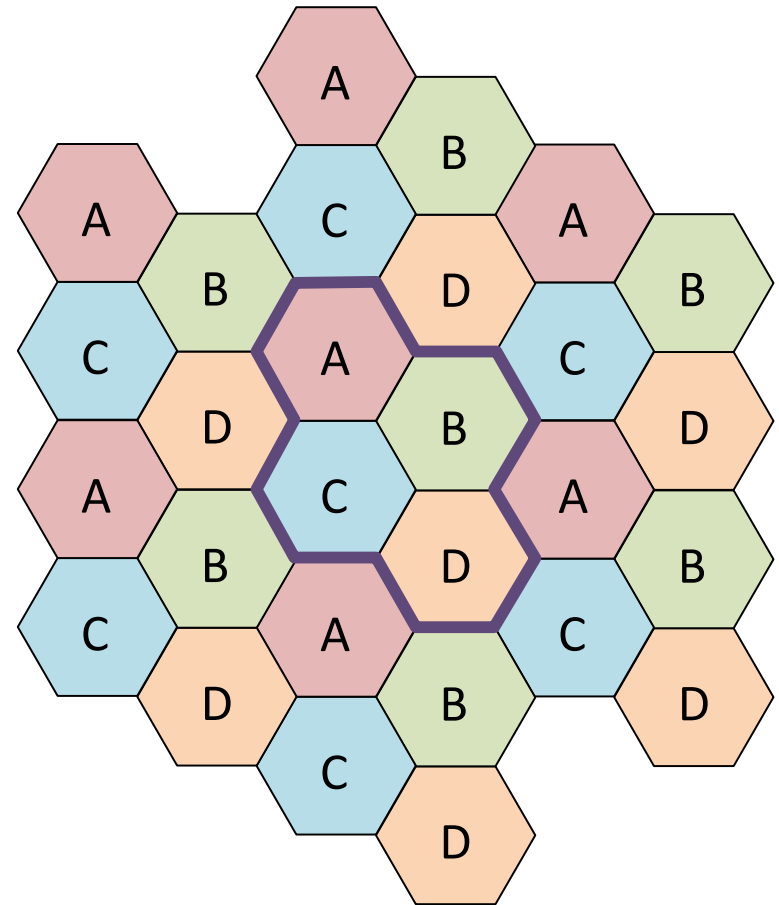**BKD 3601-7**
Monday   9:20-10:20
Wednesday 9:20-10:20

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Co-Channel Interference

- Frequency reuse → co-channel interference

- Consider only nearby interferers.
  - Power decreases rapidly as the distance increases.

- In a **fully equipped hexagonal-shaped** cellular system, there are always K = 6 cochannel interfering cells in the **first tier**.

Three Measures of Signal Quality

- Old (**noise-limited** systems) $\mathrm{SNR} = \dfrac{P_r}{P_{\mathrm{noise}}}$

- Consider both noise & interference $\quad \mathrm{SINR} = \dfrac{P_r}{P_{\mathrm{interference}} + P_{\mathrm{noise}}}$

- The best cellular system design places users that share the same channel at a separation distance (as close as possible) Why? where the intercell interference is just below the maximum tolerable level for the required data rate and BER.

- Good cellular system designs are interference-limited, meaning that the interference power is much larger than the noise power.

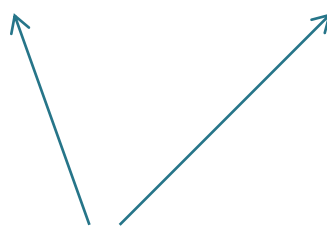$$\mathrm{SIR} = \dfrac{P_r}{P_{\mathrm{interference}}}$$

"Reliable"/"tolerable"?

(Why not as far as possible?)

Co-channel cells, must be spaced **far enough** apart so that interference between users in co-channel cells does not degrade **signal quality** below tolerable levels.

Subjective tests found that people regard an FM signal using a 30 kHz channel bandwidth to be clear if the signal power is at least **sixty times** higher than the noise/interference power.

[Klemens, 2010, p 54]

$$10\log_{10} 60 = 17.78 \approx 18 \text{ dB}$$

We will soon revisit and use these numbers for some more specific calculations

Review: Simplified Path Loss Model

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

$$P_r = \frac{P_t K d_0^{\gamma}}{d^{\gamma}} \propto \frac{1}{d^{\gamma}}$$

Captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

- K is a unitless constant which depends on the antenna characteristics and the average channel attenuation

- d_0 is a reference distance for the antenna far-field
  - Typically 1-10 m indoors and 10-100 m outdoors.

- γ is the **path loss exponent**.
  - 2 in free-space model
  - 4 in two-ray model [Goldsmith, 2005, eq. 2.17]

Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

[Goldsmith, 2005, Table 2.2]

SIR (S/I): Definition/Calculation

- $K = \#$ co-channel interfering cells

- The **signal-to-interference ratio** (S/I or SIR) for a mobile receiver which monitors a forward channel can be expressed as

$$\text{SIR} = \frac{P_r}{P_{\text{interference}}} = \frac{P_r}{\displaystyle\sum_{i=1}^{K} P_{\text{of the } i^{th} \text{ interferer}}}$$

Signal

- P_r = the desired signal **power** from the desired base station

- P_i = the interference **power** caused by the ith interfering co-channel cell base station.

- Often called the **carrier-to-interference ratio**: CIR.

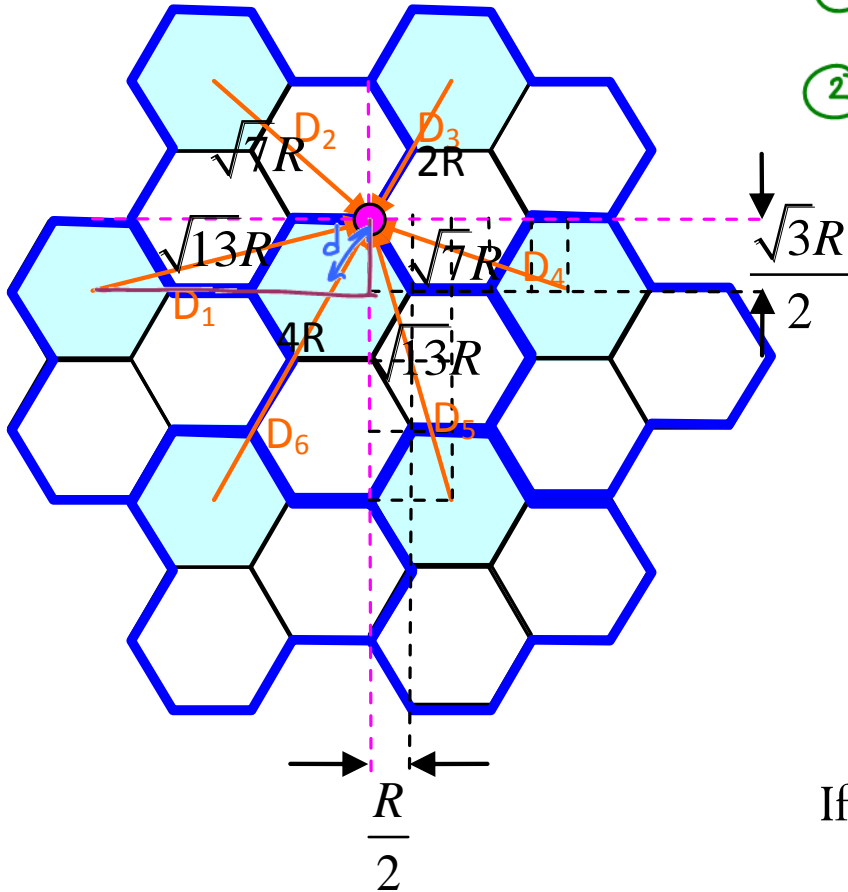[Rappaport, 2002]

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SIR Threshold

- The SIR should be greater than a specified threshold for proper signal operation.

- In the 1G **AMPS** system, designed for **voice** calls, the threshold for acceptable voice quality is SIR equal to **18 dB**.

- For the 2G digital AMPS system (D-AMPS or IS-54/136), a threshold of 14 dB is deemed suitable.

- For the **GSM** system, a range of **7–12 dB**, depending on the study done, is suggested as the appropriate threshold.

- The probability of error in a digital system depends on the choice of this threshold as well.

SIR: N = 3

Approx.

(Ignore co-channel cells that are too far away)
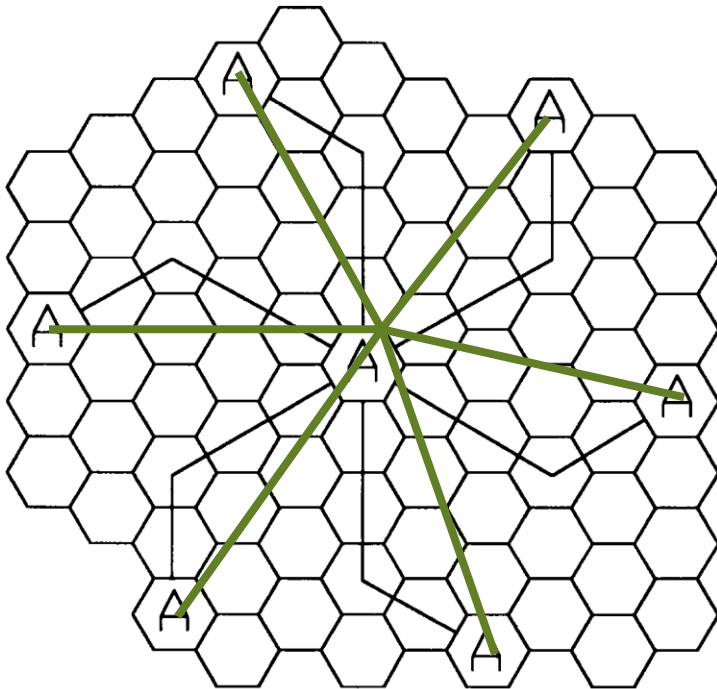
① Consider only first tier.

② Worse-case distance

d = distance btw MS & BS

d = R

$$\text{SIR} \approx \frac{k/R^{\gamma}}{\sum_i k/D_i^{\gamma}} = \frac{1}{\sum_i 1/\left(\dfrac{D_i}{R}\right)^{\gamma}} = \frac{1}{\sum_i \left(\dfrac{D_i}{R}\right)^{-\gamma}}$$

$$= \frac{1}{2\left(\sqrt{7}\right)^{-\gamma} + 2\left(\sqrt{13}\right)^{-\gamma} + 2^{-\gamma} + 4^{-\gamma}}$$

If N = 19, will the SIR be better or worse?

In the figure: D_2, $\sqrt{7}R$, D_3, $2R$, $\sqrt{13}R$, D_1, $\sqrt{7}R$, D_4, d, $4R$, $\sqrt{3}R$, D_6, D_5, $\dfrac{\sqrt{3}R}{2}$, $\dfrac{R}{2}$

Approximation

- Consider only first tier.

- Worse-case distance

$$\text{SIR} \approx \frac{1}{\displaystyle\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}}$$

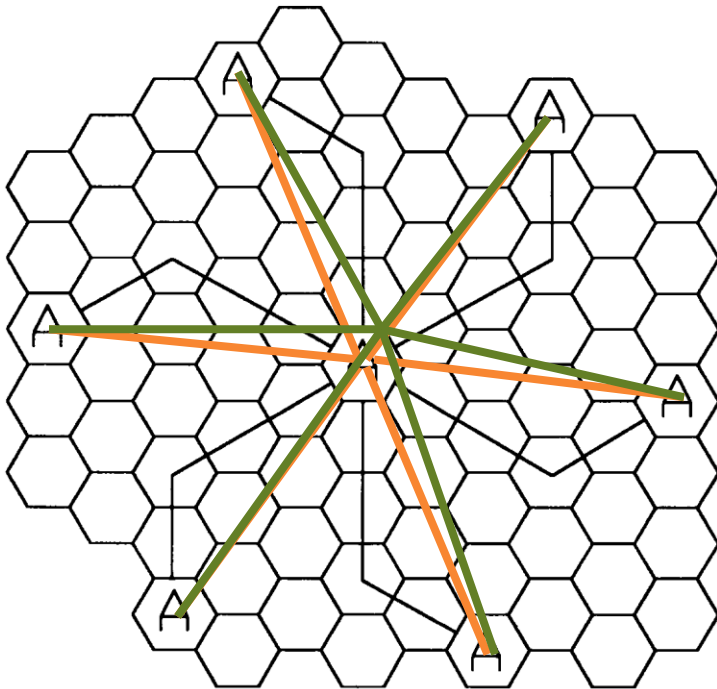- Use the same D for D_i

Approximation

- Consider only first tier.

- Worse-case distance

$$\text{SIR} \approx \frac{1}{\sum_i \left(\dfrac{D_i}{R}\right)^{-\gamma}}$$
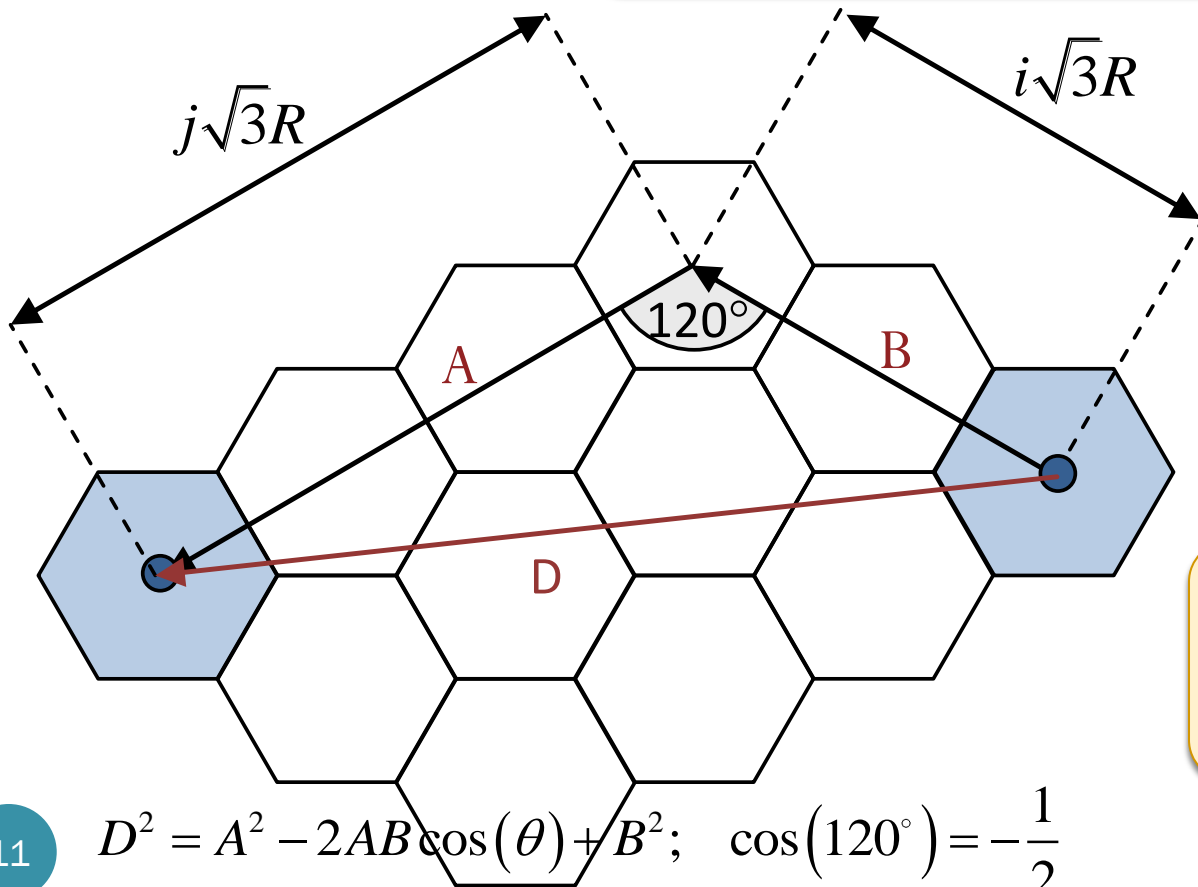
③

- Use the same D for D_i

$$\text{SIR} \approx \frac{1}{\sum_i \left(\dfrac{D}{R}\right)^{-\gamma}} \approx \frac{1}{K\left(\dfrac{D}{R}\right)^{-\gamma}} = \frac{1}{K}\left(\dfrac{D}{R}\right)^{\gamma}$$

Notice that D/R is an important quantity!

Center-to-center distance (D)

$$D = \sqrt{\left(i\sqrt{3}R\right)^2 + \left(j\sqrt{3}R\right)^2 - 2\left(i\sqrt{3}R\right)\left(j\sqrt{3}R\right)\cos\left(120°\right)}$$

$$= R\sqrt{3\left(i^2 + j^2 + ij\right)} = R\sqrt{3N}$$

$j\sqrt{3}R$

$i\sqrt{3}R$

$120°$

A

B

This distance, D, is called **reuse distance**.

D

Co-channel reuse ratio
$$Q = \frac{D}{R} = \sqrt{3N}.$$

$$D^2 = A^2 - 2AB\cos\left(\theta\right) + B^2; \quad \cos\left(120°\right) = -\frac{1}{2}$$

Q and N

> **Co-channel reuse ratio**
> $$Q = \frac{D}{R} = \sqrt{3N}.$$

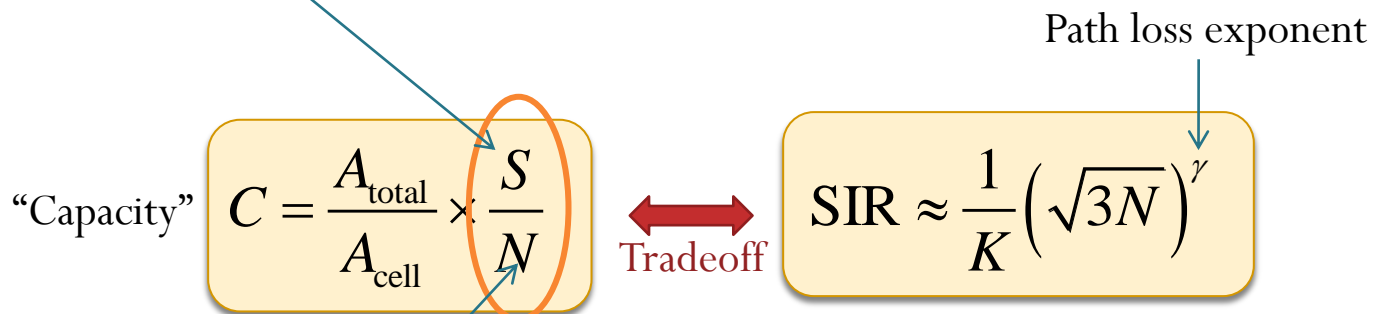|	Cluster Size (N)	Co-channel Reuse Ratio (Q)
$i = 1, \; j = 1$	3	3
$i = 1, \; j = 2$	7	4.58
$i = 0, \; j = 3$	9	5.20
$i = 2, \; j = 2$	12	6

Approximation: Crude formula
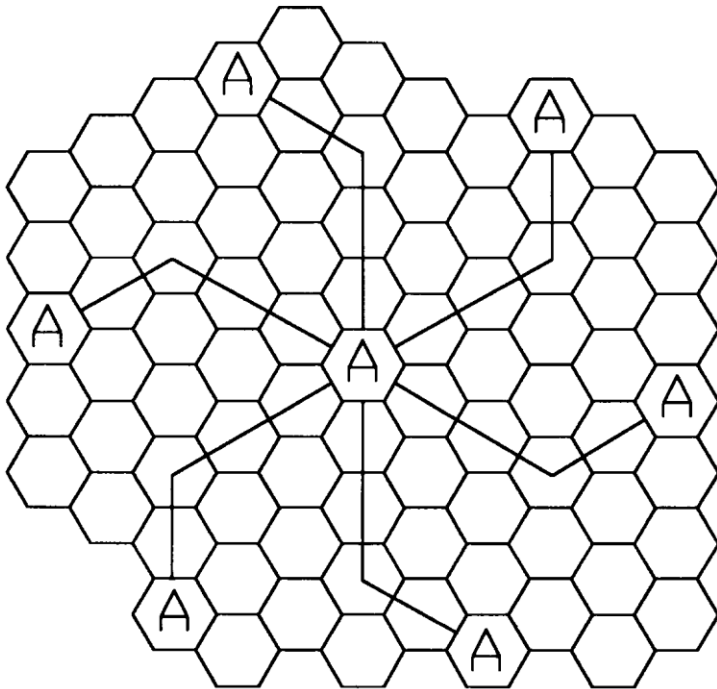
$$\text{SIR} = \frac{P_r}{P_{\text{interference}}} = \frac{P_r}{\sum_{i=1}^{K} P_{\text{of the } i^{th} \text{ interferer}}}$$

$$\approx \frac{1}{\sum_i \left(\frac{D_i}{R} \right)^{-\gamma}} \approx \frac{1}{K \left(\frac{D}{R} \right)^{-\gamma}} = \frac{1}{K} \left(\frac{D}{R} \right)^{\gamma}$$

$$= \frac{1}{K} \left(\sqrt{3N} \right)^{\gamma}$$

As the cell cluster size (N) increases, the spacing (D) between interfering cells increases, reducing the interference.

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Summary: Quantity vs Quality

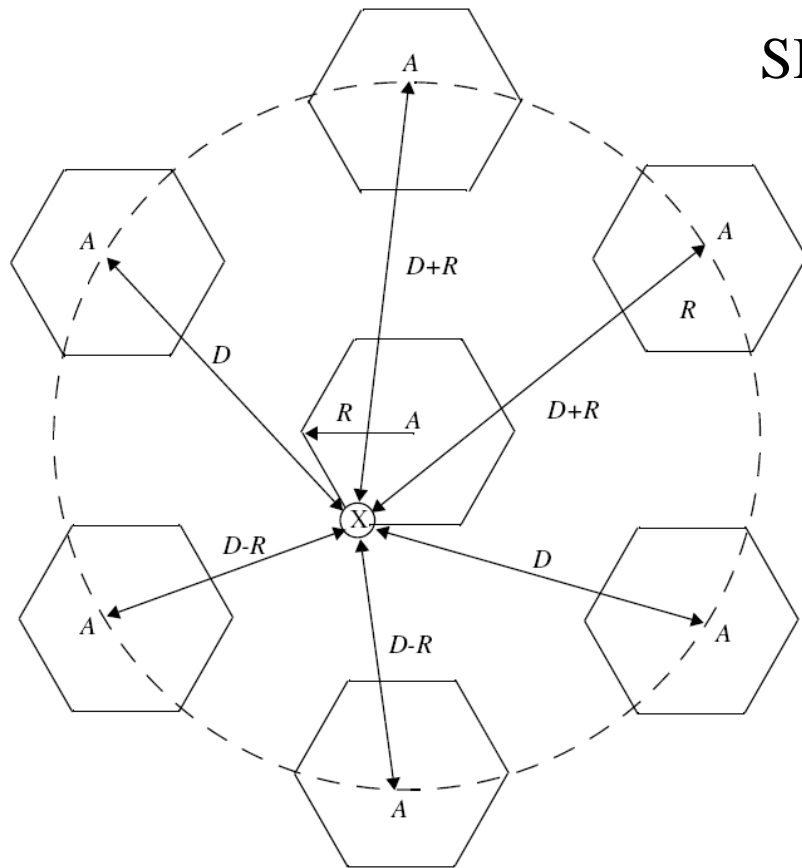S = total # available duplex radio channels for the system

Path loss exponent

"Capacity"

$$C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$$

⟷ Tradeoff

$$\text{SIR} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

m = # channels allocated to each cell.

This is only ½ of the story.

Frequency reuse with **cluster size N**

SIR: N = 7

Better approximation…

$$\text{SIR} \approx \frac{R^{-\gamma}}{2(D-R)^{-\gamma} + 2(D+R)^{-\gamma} + 2D^{-\gamma}}$$

$$= \frac{1}{2(Q-1)^{-\gamma} + 2(Q+1)^{-\gamma} + 2Q^{-\gamma}}$$

Again, $Q = \dfrac{D}{R} = \sqrt{3N}$.